

Mathematical Modelling for the Nonlinear Behavior of a Blood Vessel

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Abstract—Increasing stress and the reduction in thickness due to the localized dilation of blood vessel, especially in arteries are very important for the health of patients since it could cause sudden rupture and bleeding. High stress and weakness in the blood vessel wall due to a serious disease may end up with the rupture of cerebral aneurism which is irreversible dilatation of an artery. As it is known, a blood vessel consists of mainly three layers which are the tunica intima, the tunica media and the tunica adventitia. There are the cases that the tunica media could be broken due to the deformation and all stresses are taken only by two layers which are the tunica intima and the tunica adventitia. Therefore it is important to determine the stresses in the wall of blood vessel and the thickness reduced as a result of the deformation in the blood vessel. Large deflection effects are important for the true behavior of the blood vessels since the layers are very thin. They need to be analyzed considering the large deflection theory of elasticity. Therefore, the equilibrium equations governing their behavior are geometrically nonlinear.

Here, a blood vessel cross-section assumed to be circular in plane is considered and the behavior of a blood vessel under pressure is investigated. First, total potential energy is written as the summation of membrane and force potential energies in terms of displacements, and then by using variational principles and minimizing the total potential energy considering displacements, the nonlinear partial differential equations which govern the behavior of the blood vessels consisting of three layers are derived. The circumferential and the transverse deformations in layers are considered. Coupled partial differential equations obtained through variational principle for lateral and circumferential displacements are first discretized by central finite difference method and then solved numerically by employing iterative procedure to obtain the unknown quantities of mechanics. To observe the behavior, force-deformation graphs, maximum stresses and displacements are plotted.

Ref: Matsumoto, T; Sato, M, Analysis of stress and strain distribution in the artery wall consisted of layers with different elastic modulus and opening angle, JSME International Journal, Series C, Vol. 45, No. 4, 2002, pp. 906-912.